SOME CORRECTIONS

JONATHAN A. HILLMAN

Abstract. Here follow some corrections and additional comments for published papers of mine. (17 September 2018).

1. seifert fibre spaces and poincaré duality groups

There is a gap in the claim re HNN extensions. This gap remained in [2KG], but the proof was corrected in [ACGM].

2. Unknotting orientable surfaces . . . (with A. Kawauchi)

This paper is now in doubt.

3. simple 4-knots

The proof of Lemma 3 is not correct, and so the sufficiency of the invariants proposed later in the paper is moot.

page 917, line -7: "$t-1)\lambda$" should be "$(t-1)\lambda$".

4. deficiencies of lattices

The main argument needed cleaning up.

Theorem 1. *Let* Γ *be a finitely presentable group with a nontrivial finite normal subgroup* N *such that* Γ/N *is a lattice in* $\mathrm{PSL}(2,\mathbb{R})$*. Then* $\mathrm{def}(\Gamma)$ *is nonpositive.*

Proof. A group has a presentation of positive deficiency if and only if it is the fundamental group of a finite 2-complex with nonpositive Euler characteristic. The latter property is clearly inherited by subgroups of finite index. In particular, we may assume that Γ/N is torsion free.

Let P be a cyclic subgroup of N of prime order p, and let $G = C_\Gamma(P)$. Then $A = G \cap N$ is a central subgroup of G containing P. Moreover $[\Gamma : G] < \infty$ and so G/A is again a torsion free lattice in $\mathrm{PSL}(2,\mathbb{R})$. Hence G/A is either a nontrivial free group or is the fundamental group of an aspherical closed surface. If G/A is free then $G \cong (G/A) \times A$. If G/A is a surface group it has a subgroup H of index $|A|$, and the class in $H^2(G/A; A)$ corresponding to the central extension $1 \to A \to G \to G/A \to 1$ has image 0 in $H^2(H; A)$. Therefore the preimage of H in G splits as a direct product $H \times A$, and so Γ has a subgroup $D \cong H \times P$ of finite index. Let p be the order of P. The deficiency of D is at most $\beta_1(D;\mathbb{F}_p) - \beta_2(D;\mathbb{F}_p) = -\beta_2(H;\mathbb{F}_p)$, and so $\mathrm{def}(D) \leq 0$. Therefore $\mathrm{def}(\Gamma) \leq 0$, by the observation in the first paragraph of this proof. □

5. HOMOMORPHISMS OF NONZERO DEGREE

page 339, line 21, and Corollary 3: delete "(5,4c)".

In Lemma 5 the reference [19] should perhaps be to [St65] instead.

In the paragraph before Lemma 6. I think this is OK only if there are no reflector curves. However I think Theorem 2 remains OK – check this! make a separate argument for pure reflector curves: if $F(r)$ surjects onto σ and $2-2g \leq 1-r$ get a contradiction.

[Killing all corner points and reflectors leaves a bounded surface.]

6. CENTRALIZERS AND NORMALIZERS ...

Lemma 6 is just Corollary 8.6 of [Bieri]!

In Theorem 10 it suffices to assume that N is finitely generated. For then $\beta_1^{(2)}(G)=0$, and so $\chi(G)\geq 0$. Hence $\chi(G)=0$, and so N is FP_2, by Kochloukova in CMH 81 (2006).

7. PRO-p LINK GROUPS . . . (WITH D. MATEI AND M. MORISHITA)

The proof of Theorem 1.2.1 asserting that link groups are good (my contribution) is wrong. There is an implicit assumption that $H^2(C)\to H^2(G)$ is injective (i.e. $d_2^{0,1}=0$) in the argument for $d_3^{2,2}\gamma_3^{0,2}=\gamma_3^{2,1}d_3^{0,2}$. In fact $d_2^{0,1}$ is surjective. (The result has since been proven for the groups of "p-primitive" links, by Blomer, Linnell and Schick PAMS 2007+.)

8. STRONGLY MINIMAL PD_4-COMPLEXES

Theorem 4 is wrong. It asserted that if Z is a strongly minimal PD_4-complex, such that $\pi=\pi_1(Z)$ has one end, $v.c.d.\pi=2$ and $E^2\mathbb{Z}$ is free abelian and π has nontrivial torsion then $\pi\cong\kappa\rtimes(\mathbb{Z}/2\mathbb{Z})$, where κ is a PD_2-group.

However, if N is a connected sum of two lens spaces (other than $RP3=L(2,1)$) then $Z=S^1\times N$ is a counterexample. (The mistake was in assuming that π acts π-linearly on $\Pi=\pi_2(Z)$.) This invalidates the proof of the Corollary, but the theorem is not used elsewhere in this paper.

(This paper has been absorbed into *PD_4-complexes and 2-dimensional duality groups*, 2019 MATRIX Proceedings, 53 pp.)

9. GEOMETRIC DECOMPOSITIONS OF 4-DIMENSIONAL ORBIFOLD BUNDLES

Theorem 7 is wrong. In fact the example at end of §3 is a counter-example to this theorem!

The error seems to flow from the fifth sentence of the proof:

"Projection onto the second factor induces an orbifold bundle $p_N : N\to B$ with general fibre a closed surface and monodromy of finite order."

In fact the general fibre is a quotient of F, and so has a boundary.

10. INDECOMPOSABLE PD_3-COMPLEXES

In the first paragraph of the Introduction, the Sphere theorem and Crisp's Theorem each assume the 3-manifold or PD_3-complex to be orientable.

In Theorem 3.1, define DC_* by $DC_q=\overline{Hom_{\mathbb{Z}[\pi]}(C_{3-q},\mathbb{Z}[\pi])}$, for all q.

The second half of the account of part of Turaev's Realization Theorem in Theorem 3.2 is badly garbled. In particular, $\chi(L \cup_f e^3) = 0$ only if $\chi(K) = 1$. Fortunately, the constructive aspects of the paper involve groups with balanced presentations, and use Theorem 3.1 directly. (I think I had had this application in mind when atttempting to simplify Turaev's account!) Thus the main results are unaffected.

This section should be replaced by the following two paragraphs, which repeat Turaev's construction, but differ slightly in the verification that it works.

Conversely, let K be the finite 2-complex associated to a presentation for π, and let M be the corresponding Fox-Lyndon presentation matrix for I_π. Suppose first that $J_\pi \oplus \mathbb{Z}[\pi]^m \cong I_\pi \oplus \mathbb{Z}[\pi]^n$. Let $L = K \vee mS^2$. Then $\pi_1(L) \cong \pi_1(K)$ and $\mathrm{Cok}(\partial_2^L) \cong I_\pi \oplus \mathbb{Z}[\pi]^n$. Let $C_* = C_*(\widetilde{L})$ and let $\alpha : DC_1 \to \mathbb{Z}[\pi]$ be the composite of the projection onto $J_\pi \oplus \mathbb{Z}[\pi]^m$, the isomorphism with $I_\pi \oplus \mathbb{Z}[\pi]^n$, and the inclusion into $\mathbb{Z}[\pi]^{n+1}$. Then $\bar{\alpha}^{tr} : \mathbb{Z}[\pi]^{n+1} \to C_2$ has image in $\pi_2(L) = H_2(C_*)$. Let $\{e_1, \dots, e_{n+1}\}$ be the standard basis of $\mathbb{Z}[\pi]^{n+1}$, and let f_i be a map in the homotopy class of $\bar{\alpha}^{tr}(e_i)$, for $i \leq n+1$. Let X be the 3-complex obtained by adjoining 3-cells to L along the $n+1$ maps $\{f_1, \dots, f_{n+1}\}$.

The arguments of Theorem 3.1 apply to X, as we now show. Clearly $H_0(C_*) = H_0(DC_*) = \mathbb{Z}$. If π is finite then $H^0(C_*) \cong H^0(DC^*) \cong \mathbb{Z}$ and $H_1(C_*) = H_1(DC_*) = H^1(C^*) = H^1(DC^*) = 0$. Hence $\widetilde{X} \simeq S^3$ and so X is a PD_3-complex. If π is infinite then $H_1(C_*) = H_1(DC_*) = 0$, $H^1(C^*) \cong H^1(CD^*) \cong e^1\mathbb{Z} = Ext^1_{\mathbb{Z}[\pi]}(\mathbb{Z}, \mathbb{Z}[\pi])$ and $H^0(C^*) = H^0(DC^*) = 0$, so $H_3(C_*) = H_3(DC_*) = 0$. Let $x \in C_3$ represent a generator of $H_3(X; \mathbb{Z}^w)$. Then slant product with x induces a chain homomorphism θ from DC_* to C_* which induces isomorphisms in degrees 0 and 1. Hence it induces isomorphisms $H^1(C^*) \cong H^1(DC^*)$, and so $H_2(\theta) : H_2(DC_*) = H^1(C^*) \to H_2(C_*) = H^1(DC^*)$ is an isomorphism. Since $H_i(\theta)$ is an isomorphism for all i and C_* and DC_* are projective chain complexes, θ is a chain homotopy equivalence. Hence X is a finite PD_3-complex with fundamental group π.

My surname is misspelt on page 135, line -8; this was clearly an intervention by someone (with first language German?) expanding a reference to give the authors' names in the text, in line with the house style, and I overlooked this in reading final proofs.

In the proof of Theorem 5.2, the third sentence of the first paragraph should be "If 4 divides $|G_e|$ then G_e has a central involution, which is also central in $V = G_{o(e)}$ and $W = G_{t(e)}$, since these groups have cohomological period 4. (See the remarks preceding Lemma 2.1.)"

In the third paragraph, d should be k, say, as it is NOT the d of the statement, and the final sentence should be "Since the odd-order subgroup of G_e is central in W its normalizer in V must be abelian. Hence either $k = 3$ and $V = B \times Z/dZ$ with $B = T_1^*$ or I^* and $(d, |B|) = 1$ or $k = 1$, by Lemma 5.1."

Minor improvements have been made to the next two statements, to emphasize the hypothesis that π is virtually free.

Lemma (7.3)**.** *Let X be an indecomposable PD_3-complex with $\pi = \pi_1(X) \cong F(r) \rtimes G$, where G is finite. If π has an orientation reversing element g of finite order then G has order $2m$, for some odd m, and so π has an orientation reversing involution.* □

In the first paragraph of the proof of Lemma 7.3 we may and should also reduce to the case when G is abelian, of order a multiple of 4. (See *Indecomposable non-orientable PD_3-complexes*, Alg. Geom. Top. 17 (2017), 645–656.) The rest of the proof then goes through as written.

Theorem (7.4). *Let X be an indecomposable, non-orientable PD_3-complex with $\pi = \pi_1(X)$ virtually free. If π has an orientation reversing involution then $X \simeq S^1 \times RP^2$.* □

As the statement of Theorem 7.4 now assumes that π is virtually free, the first sentence of the proof should be deleted. In the final sentence of the second paragraph of the proof, the vertex groups must all be D_{2p}, by the normalizer condition and Crisp's Theorem (and so $\varepsilon = -1$, later in the proof). In the presentation, a_i should be a (no subscript). The argument otherwise needs no change.

Corollary 7.5 only follows from Theorem 7.4 if π is virtually free. However it holds in general. (See *Indecomposable non-orientable PD_3-complexes*, loc.cit.), In fact, Crisp's result already implies that $C_\pi(g) \cong \langle g\rangle \times \mathbb{Z}$ or $\langle g\rangle \times D_\infty$, since every element of the maximal finite normal subgroup of a group with two ends has infinite centralizer.)

Finally, some typos:
Abstract: "$\mathbb{Z}6\mathbb{Z}$" should be "$\mathbb{Z}/6\mathbb{Z}$".
Theorem 3.2, second paragraph of proof: "fthat" should be "that".
Statement of Theorem 5.2: "$\mathbb{Z}2\mathbb{Z}$" should be "$\mathbb{Z}/2\mathbb{Z}$".
Second paragraph of §6: b_1 should be b_n in the last relator of the presentation.

11. 2-knots with solvable groups

on page 989:
line 4: "$t = -(m+n)e$" should be "$t = (m+n)e$".
line 15: delete "=" after "$Aut(\Gamma(e,1))$".
line 17: the final relation should be "$rk = kr$".
line 20: the final relations should be "$rc_ur = c_v^{-1}, rc_vr = c_u^{-1}$".
line -3 (first para of §12): "$(I_2 + \beta^{-1})$" should be "$(I_2 + \beta)^{-1}$".

12. Seifert fibred knot manifolds

The internal referencing of results needs correction in numerous places.
page 7, line 21: $\pi^{orb}\mathbb{D}(3,\overline{2})$ and $\pi^{orb}\mathbb{D}(\overline{2},\overline{3},\overline{5})$ are finite.
page 8, line 13: "> 1" should be "> 2".
page 8, line -12: The groups $\pi^{orb}\mathbb{D}(a,b)$ all have weight 1, since adjoining the relation $v = xw$ kills the group. Thus the first open cases have presentations

$$\langle v, w, x, y \mid v^a = w^b = x^2 = (xy)^d = 1,\ yvw = vwx\rangle.$$

page 13, lines 6-7: the assertion on this line has not been proven.
page 13, line 12: add " and the homology sphere is S^4".
Further on, "$\langle z\rangle$" should be "$\langle y^{-1}z\rangle$".
At the end of the first sentence of §6,
"$S(2,3,4)$ or $S(2,3,5)$" should be "$S(2,3,4)$, $S(2,3,5)$ or "$\mathbb{D}(3,\overline{2})$".
biblio: [Gabai]; the journal should be *J. Diff. Geometry.*
If $w(G * \mathbb{Z}) > w(G)$ then we may exclude $P(c_1, c_2, c_3)$. For if G has presentation $\langle v_1, v_2, v_3 | v_1^{c_1} = v_2^{c_2} = v_3^{c_3} = 1\rangle$ then $w(G) = 2$. Since the group π with presentation

$\langle u, v_1, v_2, v_3 | v_i^{c_i} = 1,\ u^2 = v_1 v_2 v_3 \rangle$ is obtained from $G * \mathbb{Z}$ by adjoining one relation, $w(\pi) \geq w(G * \mathbb{Z}) - 1$.

The hypothesis "$w(G*\mathbb{Z}) > w(G)$" implies the Kervaire-Laudenbach Conjecture. However it is very plausible, and so there is some reason to feel that the possible non-orientable Seifert bases are known.

13. THE $\mathbb{F}_2$-COHOMOLOGY RINGS OF $\mathbb{S}\mathrm{ol}^3$-MANIFOLDS

page 199, line -4: "$BU(M, \phi) = 2$ if and only if $\phi^2 = 0$" should be "$BU(N, \phi) = 2$ if and only if $\phi^3 = 0$".

14. COMPLEMENTS OF CONNECTED HYPERSURFACES IN S^4

In paragraph 3 of §3 it should be assumed that L is bipartedly *trivial*, rather than just bipartedly slice.

In the statement of Theorem 7.1, the condition "If $\pi_1(X)$ is abelian" should be imposed in the second sentence, rather than in the final sentence.

On the final page, the two sentences relating to the circle bundles over the Klein bottle ($M(-2; (1, 0))$ and $M(-2; (1, 4))$) should be deleted, as they were based on a misreading of Kreck's paper.

15. SOLVABLE NORMAL SUBGROUPS OF 2-KNOT GROUPS

The argument for the key Lemma 3.2 now seems inadequate. There is a near counter-example; it fails for $G = A \rtimes \mathbb{Z}$, where A is the elementary abelian group of exponent p on generators a_n, and $\mathbb{Z}$ acts by translating the indices. (This group is not FP_2.) The lemma remains true when T is an increasing union of finite normal subgroups, but this is only enough to establish the result on centres: eother π' is finite (of even order) or $\zeta\pi$ is torsion-free. Thus most of the results of the first half of the paper are moot. (However I have no counter example to these results.) The result on centres is OK.

(The main result holds if the solvable knot group is almost coherent or if it has an abelian normal subgroup of rank > 0, in particular, if it is torsion-free. See Chapter 15 of [FMGK].)

16. DEFICIENCY AND COMMENSURATORS

page 514, line 3: the sentence should read "the kernel of a homomorphism between finitely generated free R-modules. Hence ... R is a noetherian UFD."

page 521, line 8: "$r_k(\beta)$" should be "$r_j(\beta)$".

page 522: the group with presentation

$$\langle t, x \mid t^4 x t^{-4} = t^2 x^2 t^{-1} x^{-1} t^{-1} x^{-1},\ x t^2 x t^{-2} = t^2 x t^{-2} x \rangle$$

has Hirsch length 5 and deficiency 0. Thus it is a counterexample to the estimate $def(G) \leq 2 - \lfloor \frac{n+1}{2} \rfloor$ suggested for poly-Z groups of Hirsch length n. Does this bound hold for torsion-free nilpotent groups?

References

[1] Bieri, R.
[2] Blomer, I.,
[3] Hillman, J. A.
[4] Hillman, J. A.
[5] Hillman, J. A.
[6] Stallings, J. Homology and central series of groups, J. Algebra 2 (1965), 170–181.

School of Mathematics and Statistics, University of Sydney, NSW 2006, Australia
Email address: jonathan.hillman@sydney.edu.au