Errata and Addenda for the current (2021) revision of "Four-Manifolds, Geometries and Knots"

31 May 2021

The main changes in 2020 were the removal of the penultimate section of Chapter 12, as there was an error in the argument showing that there are exactly four homeomorophism types of closed 4-manifolds M with $\pi_1(M) = Z/4Z$ and $\chi(M) = 1$, and the rewriting of Chapter 15. (The changes to Chapter 12 affect also the preprint [HH17], which is being rewritten.)

Chapter 2:

page 27, line 7: the "Euler characteristic formula" is $\chi(X) = \Sigma(-1)^i \beta_i^{(2)}(X)$, of course.

page 34, line 15: This sentence could be expanded out as follows:

If X and Y are PD_n-complexes and Y is non-orientable then a map $f : X \to Y$ such that $f^* w_1(Y) = w_1(X)$ only determines a homomorphism from $H_n(X; \mathbb{Z}^{w_1(X)})$ to $H_n(Y; \mathbb{Z}^{w_1(Y)})$ *up to sign*, as one must choose a lift $f^+ : X^+ \to Y^+$. A similar issue arises with other non-constant local coefficient systems. (See [Ta08] for a thorough discussion of the subtleties here. This oversight has no serious consequences for the present work.)

page 36, Theorem 2.11: the relevant action used here is the natural right action on $Ext^1_{\mathbb{Z}[\pi]}(\mathbb{Z}, \mathbb{Z}[\pi])$, NOT the action via conjugation considered in [Br:Chapter III.§8].

page 44, middle: the identification of $k_1(X)$ with an iterated extension class deserves comment, as I have not found a published proof, although I suspect one may be found in the work of Eilenberg, Mac Lane and J.H.C. Whitehead of the late 1940s. nd so represents the class of the extension.

If P is aspherical no additive natural transformation takes $k_1(P) = 0$ to $[P]$.

Chapter 3

page 58, line 2: the hypothesis "$H^4(\pi; F) = 0$" can be weakened to "all cup products of pairs of elements of $H^2(\pi; F)$ are trivial", without change to the proof.

page 62, line 1: $q^{SG}(\pi)$ has been determined for π a 3-manifold group with no 2-torsion [SW21].

Chapter 5

page 91, Theorem 5.4 (1): insert "non-trivial" before "finitely generated".

Chapter 11:

The groups $P''_{48.3^{k-1}a} \times Z/dZ$ were overlooked here! (This oversight has no consequences for other chapters. In particular, these groups cannot be knot commutator subgroups, as they have abelianization $Z/2dZ$.)

page 219, line 4: insert "the groups P''_{48r} (with r odd > 1)," after "O^*_k,".

page 222, line 5 (immediately after the treatment of O^*_k): insert

$$Z/aZ \rtimes_{-1} O^*_k = \langle O^*_k, u \mid u^a = 1,\ wuw^{-1} = u^{-1},\ x, y, z \leftrightharpoons u \rangle$$

Here $(a, 6) = 1$. This is the group $P''_{48.3^{k-1}a}$ of [Mi57]. An automorphism of this group induces automorphisms of the normal subgroup Z/aZ and of the quotient O^*_k. It is easily seen that $Aut(P''_{48.3^{k-1}a}) \cong Aut(Z/aZ) \times Aut(O^*_k)$. Since conjugation by w induces the inversion on Z/aZ, $Out(P''_{48.3^{k-1}a}) \cong Aut(Z/aZ) \rtimes_{-1} Out(O^*_k)$. Since $J(O^*_k) = 1$, we have $J(P''_{48.3^{k-1}a}) \cong J_+(Z/aZ)$.

Chapter 12:

(30 January 2020) The penultimate section, on the case $\pi = Z/4Z$, has been deleted as there was a crucial error in identifying the action of π on the groups $H^q(K(\Pi, 2); \pi_3(M))$.

Chapter 15:

page 299: "$g \in \pi^+$" should be "$w_1(X)(g) = 1$", as the notation π^+ has not been defined.
page 306, line -4: "15.14" should be "15.12".

Chapter 16:

page 323, lines -3 and -2: the sentences on these lines should read

"the kernel of a map between finitely generated free R-modules. Hence $H_2(X; R) \cong R$, as it has rank 1 and R is a noetherian UFD."

(Note also that if $p \neq 0$ then $\Lambda/p\Lambda$ is a PID, while Λ has global dimension 2 and all projective Λ-modules are free.)

Chapter 18:

page 364, line -4: "Chapter 12" should be "Chapters 8, 12 and 13".

Bibliography: The following list includes the items cited from the arXiv in the 2018 revision (but not yet published elsewheres) and new references. The details shall be updated when possible.

References

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[HH14] *is now* [HH17] Hambleton, I. and Hillman, J.A. Quotients of $S^2 \times S^2$, arXiv: 1712.04572 [math.GT]

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[Hi13b] Hillman, J.A. Finiteness conditions in covers of Poincaré duality spaces, arXiv: 1301.3972 [math.GT].

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