Errata for "Iwahori–Hecke algebras and Schur algebras of the symmetric group"

Chapter 1

Page 9, Corollary 1.17: $\mathcal{K}$ should be the algebraic closure of $\mathbb{C}(\hat{q})$

Chapter 2

Page 17, line -2: Replace "...and one can check that $C^{*\lambda} \cong \operatorname{Hom}_R(C^\lambda, R)$" with "and $\operatorname{Hom}_R(C^{*\lambda}, R)$ is a right A–module with A–action given by $(f \cdot a)(x) = f(ax)$, for $f \in \operatorname{Hom}_R(C^{*\lambda}, R)$, $a \in A$ and $x \in C^{*\lambda}$).

Page 21, proof of 2.18: Delete "that $C^\nu \cong \operatorname{Hom}_R(C^{*\nu}, R)$ as right A–modules. Therefore" and after the displayed equation add "Hence, $\dim P^\lambda \otimes_A C^{*\nu} = [\operatorname{Hom}_R(C^{*\nu}, R) : D^\lambda] = d_{\nu\lambda}$ since $\operatorname{Hom}_R(D^{*\mu}, R) \cong D^\mu$ whenever $D^\mu \neq 0$."

Page 24, exercise 7(i). Delete "In addition, show that $C^\lambda \cong \operatorname{Hom}_R(C^{*\lambda}, R)$ for all $\lambda \in \Lambda$.

Page 25, line 3: change parenthetical remark to (the four Kazhdan–Lusztig bases $\{C_x\}$, $\{C'_x\}$, $\{D_x\}$ and $\{D'_x\}$ of $\mathscr{H}(\mathfrak{S}_n)$ are all cellular; however, this is not true for Hecke algebras of other types).

Chapter 3

Page 38, Warning: S^λ is the dual of the Dipper–James Specht module indexed by λ; that is, $S^\lambda \cong (S^\lambda_{DJ})^\diamond$. One can check that $(S^\lambda_{DJ})^\diamond \cong S^{\lambda'}$ so it is necessary to replace λ with λ' when comparing our results with those of Dipper and James.

Page 41, line -1: The reduction to the case where $k = n$ is a bit of a leap. Here are more details.

Suppose $k < n$ where k is the number in the last row and the first column of $\mathfrak{t}^\lambda$. As in the second paragraph of the proof, let $\mu = (\lambda_1, \dots, \lambda_r, 1)$. Then $k = |\mu|$ and $m_\lambda = h m_\mu$ if we set $h = \sum_w T_w$, where the sum is over the elements of $\mathfrak{S}(\{k, k+1, \dots, n\})$. By the argument for the case when $k = n$ it follows that

$$m_\lambda L_k = h m_\mu L_k = h\Big(\operatorname{res}_{\mathfrak{t}^\lambda}(k) m_\mu + \sum_{\substack{\nu \vdash k, \nu \rhd \mu \\ \mathfrak{u},\mathfrak{v} \in \operatorname{Std}(\nu)}} r_{\mathfrak{u}\mathfrak{v}} m_{\mathfrak{u}\mathfrak{v}}\Big),$$

for some $r_{\mathfrak{u}\mathfrak{v}} \in R$. If ν is a partition of k let $\hat{\nu}$ be the partiton of n obtained by appending 1^{n-k} to ν and if $\mathfrak{u} \in \operatorname{Std}(\nu)$ let $\hat{\mathfrak{u}}$ be the unique $\hat{\nu}$-tableau such that $\hat{\mathfrak{u}} \downarrow k = \mathfrak{u}$. Under the natural embedding $\mathscr{H}(\mathfrak{S}_k) \hookrightarrow \mathscr{H}(\mathfrak{S}_n)$, it is easy to see that if $\nu \rhd \mu$ and $\mathfrak{u}, \mathfrak{v} \in \operatorname{Std}(\nu)$ then $m_{\mathfrak{u}\mathfrak{v}} = m_{\hat{\mathfrak{u}}\hat{\mathfrak{v}}}$. Notice that if $\nu \rhd \mu$ then $\hat{\nu} > \lambda$. Therefore, returning to the last displayed equation,

$$m_\lambda\big(L_k - \operatorname{res}_{\mathfrak{t}^\lambda}(k)\big) = h \sum_{\substack{\nu \rhd \mu \\ \mathfrak{u},\mathfrak{v} \in \operatorname{Std}(\nu)}} r_{\mathfrak{u}\mathfrak{v}} m_{\mathfrak{u}\mathfrak{v}} \in \mathscr{H}^\lambda \cap \bigcap_{\nu > \lambda} \mathscr{H}^\nu \subseteq \check{\mathscr{H}}^\lambda.$$

Therefore, $m_\lambda L_k \equiv \operatorname{res}_{\mathfrak{t}^\lambda}(k) m_\lambda \pmod{\check{\mathscr{H}}^\lambda}$ as we needed to show.

Page 44, line -6: missing bracket

Page 46, line 11: Replace $\mathfrak{l}^\lambda_e$ with l_e throughout.

Page 46, Lemma 3.40: The proof of this lemma is not particularly clear. Here is a better argument:

Let $\mathbb{L}_0, \mathbb{L}_1, \dots, \mathbb{L}_k$ be the ladders which meet $[\lambda]$ and let r_a be the row index of the highest node in $\mathbb{L}_a \cap [\lambda]$, for $0 \le a \le k$. By definition, if $b \ge 1$ then $\lambda_b - \lambda_{b+1} < e$ because λ is e-restricted. Therefore, if (i, j) and (i', j') are two nodes in the ladder $\mathbb{L}_k$, for $k = l_e(i, j)$, then $(i', j') \in [\lambda]$ whenever $(i, j) \in [\lambda]$ and $i \le i'$. That is, the ladder $\mathbb{L}_0, \dots, \mathbb{L}_k$ are 'unbroken' in the sense that all of the nodes which belong to the same ladder as (i, j) and appear in a later row also belong to $[\lambda]$. Hence, we see that $r_0 \ge r_1 \ge \cdots \ge r_k$.

Now suppose that $\mathfrak{t}$ is a standard tableau, not necessarily of shape λ, such that $\operatorname{res}(\mathfrak{t}) = \operatorname{res}(\mathfrak{l}^\lambda_e)$. To prove the Lemma we have to show that $\mathfrak{t} \rhd \mathfrak{l}^\lambda_e$. We argue by induction on n.

When $n = 1$ then $\mathfrak{t} = \mathfrak{l}^\lambda_e = \mathfrak{t}^\lambda$ and there is nothing to prove, so suppose that $n > 1$. Let $i = \operatorname{res}_{\mathfrak{l}^\lambda_e}(n)$ and let m be maximal such that $\operatorname{res}_{\mathfrak{l}^\lambda_e}(m) \neq i$. Thus, recalling that $\mathbb{L}_0, \mathbb{L}_1, \dots, \mathbb{L}_k$ are the ladders which meet $[\lambda]$, n appears at the top of the ladder $\mathbb{L}_k$ in $\mathfrak{l}^\lambda_e$ and m appears at the top of $\mathbb{L}_{k-1}$. Let $\mu = \operatorname{Shape}(\mathfrak{l}^\lambda_e \downarrow m)$.

As noted above, the ladders $\mathbb{L}_0, \dots, \mathbb{L}_k$ are unbroken. Therefore, μ is an e-restricted partition and $\mathfrak{t}^\mu_e = \mathfrak{t}^\lambda_e{\downarrow}m$. Moreover, $\mathfrak{t}{\downarrow}m$ is a standard tableau with residue sequence $\operatorname{res}(\mathfrak{t}^\mu_e)$. Therefore, by induction on n,

$$(\dagger) \qquad \operatorname{Shape}(\mathfrak{t}{\downarrow}r) \trianglerighteq \operatorname{Shape}(\mathfrak{t}^\mu_e{\downarrow}r) = \operatorname{Shape}(\mathfrak{t}^\lambda_e{\downarrow}r),$$

for $r = 1, 2, \dots, m$, with equality throughout if and only if $\mathfrak{t}{\downarrow}m = \mathfrak{t}^\mu_e$. Consequently, if $\mathfrak{t} \ntrianglerighteq \mathfrak{t}^\lambda_e$ then there must exist an integer p such that $\operatorname{Shape}(\mathfrak{t}{\downarrow}p) \ntrianglerighteq \operatorname{Shape}(\mathfrak{t}^\lambda_e{\downarrow}p)$ and $m < p \le n$. Let p be minimal with this property and set $\nu = \operatorname{Shape}(\mathfrak{t}{\downarrow}p)$. For convenience, set $\lambda = (\lambda_1, \lambda_2, \dots)$, $\mu = (\mu_1, \mu_2, \dots)$ and $\nu = (\nu_1, \nu_2, \dots)$. By construction, p appears in row $r_k + n - p$ of $\mathfrak{t}^\lambda_e$. Therefore, by the minimality of p,

$$\nu_1 + \dots + \nu_{r_k+n-p} < \lambda_1 + \dots + \lambda_{r_k+n-p} = \mu_1 + \dots + \mu_{r_k+n-p} + 1.$$

Since $\mathfrak{t}{\downarrow}m \trianglerighteq \mathfrak{t}^\mu_e$ this forces $\nu_1 + \dots + \nu_{r_k+n-p} = \mu_1 + \dots + \mu_{r_k+n-p}$ and hence that $\mathfrak{t}{\downarrow}m = \mathfrak{t}^\mu_e$. As $m+1, \dots, n$ occupy the lowest addable i-nodes in $\mathfrak{t}^\lambda_e$ this easily implies that $\mathfrak{t} \trianglerighteq \mathfrak{t}^\lambda_e$, giving a contradiction and completing the proof.

Page 47, line 4: $\overline{\lambda^i}$ is a *composition* of λ_i.

Page 47, line 5: g_i is the greatest common divisor of $\{\, [\gamma]^!_q \in \mathcal{Z} \mid \gamma \trianglerighteq \overline{\lambda}^i \,\}$. That is, we must define g_i to be an element of the ring $\mathcal{Z} = \mathbb{Z}[q, q^{-1}]$.

Page 47, Lemma 3.42: Suppose that $R = \mathcal{Z}$, that $\lambda = (\lambda_1, \dots, \lambda_k)$ is a partition of n,...

Page 47, line -7: On the other hand, if $\lambda \trianglerighteq \nu$ then $(\lambda_1, \lambda_2, \dots, \lambda_k) \trianglerighteq (\lambda_1, \lambda_1 - \mu_1, \dots, \lambda_k - \mu_k); \dots$

Page 50, exercise 18(ii): As pointed out to me by Eric Marberg, it is not true that $\mathfrak{S}_\mu$ is a subgroup of the column stabiliser of $\mathfrak{t}^\lambda d$; for example, take $\lambda = \mu = (2, 2, 1)$ and $d = (2, 3, 4, 5)$. Instead one should prove that i and j are in different rows of $\mathfrak{t}^\lambda d$ whenever they are in the same row of $\mathfrak{t}^\mu$. This implies that $\mu' \trianglerighteq \lambda$.

Page 53, line -7: $h_i = h^\lambda_{i1}$.

Chapter 4

Page 58, line 4: $\operatorname{last}(\mathtt{T}) = \begin{array}{|c|c|c|c|}\hline 1&2&6&8\\\hline 3&4&5&7\\\hline\end{array}$.

Chapter 5

Page 70, line 5: if and only if $d(\operatorname{first}(\mathtt{S})) \trianglerighteq d(\operatorname{first}(\mathtt{T}))$.

Page 72, line 2: then $a < c$

Page 72, before Lemma 5.12: Insert the lines "Define maps $\varphi_{\mathtt{T}^\mu\mathtt{T}^\omega} : M^\mu \longrightarrow M^\omega$ and $\varphi_{\mathtt{T}^\omega\mathtt{T}^\mu} : M^\omega \longrightarrow M^\mu$ by $\varphi_{\mathtt{T}^\mu\mathtt{T}^\omega}(m_\mu h) = m_\mu h$ and $\varphi(h) = m_\mu h$ for all $h \in \mathscr{H}$. Note that $M^\omega = \mathscr{H}$. Both $\varphi_{\mathtt{T}^\mu\mathtt{T}^\omega}$ and $\varphi_{\mathtt{T}^\omega\mathtt{T}^\mu}$ are elements of the Schur algebra.

Page 73, line 18: Note that if $x = (i, j)$ and $y = (k, l)$ then $j - i + k - l + 1$.

Page 74, line 5: Replace $\mathfrak{S}_n$ with $\mathfrak{S}_\mu$.

Page 74, proof of Lemma 5.15(ii): The argument should be replaced with:

Now suppose that $\mathtt{T} = \mu(\mathfrak{t})$ is not semistandard. By definition, $\mathtt{T}$ is row semistandard so there must exist integers $i < j$ such that i and j are in the same row of $\mathfrak{t}^\mu$ and in the same column of $\mathfrak{t}$. Consequently, the tableau $\operatorname{last}(\mathtt{T})$ is row standard but not standard. As in the last paragraph, $m_\mathfrak{t} m_\mu = [\nu]^!_q m_\mathtt{V}$, where $m_\mathtt{V} = \sum_\mathfrak{v} m_\mathfrak{v}$ and the sum is over the row standard λ-tableau $\mathfrak{v}$ such that $\mu(\mathfrak{v}) = \mathtt{T}$. Now if $\mathfrak{v}$ is row standard and $\mu(\mathfrak{v}) = \mathtt{T}$ then $\mathfrak{v} \trianglerighteq \operatorname{last}(\mathtt{T})$, so if

$$m_\mathfrak{t} m_\mu = \sum_{\mathfrak{s} \in \operatorname{Std}(\lambda)} c_\mathfrak{s} m_\mathfrak{s}, \qquad \text{for } c_|s \in R,$$

then $c_\mathfrak{s} \neq 0$ only if $\mathfrak{s} \triangleright \operatorname{last}(\mathtt{T})$ by Lemma 3.15. Note that if $c_\mathfrak{s} \neq 0$ then $\mathfrak{s}$ strictly dominates $\operatorname{last}(\mathtt{T})$ because $\operatorname{last}(\mathtt{T})$ is not standard. On the other hand, by Proposition 4.14, $m_\mathfrak{t} m_\mu$ belongs to the span of $\{\, m_\mathtt{S} \mid \mathtt{S} \in \mathcal{T}_0(\lambda, \mu) \,\}$, so $c_\mathfrak{s} \neq 0$ only if $\mu(\mathfrak{s}) = \mathtt{S} \in \mathcal{T}_0(\lambda, \mu)$. It follows that $m_\mathfrak{t} m_\mu = \sum_\mathtt{S} c_\mathtt{S} m_\mathtt{S}$, where $c_\mathtt{S} \neq 0$ only if $\mathtt{S} \triangleright \mathtt{T}$. Hence, (ii) follows.

Page 75, line 1: Suppose that $\mathfrak{t} = \operatorname{first}(\mathtt{T})$. Then $\pi^\mu_\mathfrak{t} \varphi_\mathtt{T} = \varphi_\mathfrak{t} \varphi_{\mathtt{T}^\omega\mathtt{T}^\mu} \dots$

Page 78, line -2: B_4 should be G_4.

Page 78, line -2: ...columns $c_1 < c_2 < \dots < c_z$

Page 81, line 15: $M_{\mathbb{F}}(k) = M(k) \otimes_R \mathbb{F}$ should be $M_{\mathbb{F}}(k) = (M(k) + \mathfrak{p}M(k))/\mathfrak{p}M(k)$.

Page 85, line 10: *is* well–defined

Page 86, Proof of Theorem 6.37: The last paragraph of the proof contains a gap. The proof breaks down in the case where W^λ is irreducible. The following argument corrects this.

To prove the theorem it is enough to show that $W^\mu_{\mathbb{F}}$ and $W^\gamma_{\mathbb{F}}$ are in the same block. If $\nu_{\mathfrak{p}}([h^\mu_{ab}]_{\hat q}) \neq \nu_{\mathfrak{p}}([h^\mu_{ac}]_{\hat q})$, for some nodes $(a,b),(a,c) \in [\mu]$, then we may assume that $b = 1$ and let ν be the partition obtained by removing the (a,c)-hook from μ and wrapping it back on at the bottom of the first column of $\mu \backslash R^\mu_{ac}$. Then $\mu \rhd \nu$ and $W^\nu_{\mathbb{F}}$ appears with non–zero multiplicity in $\sum_{i>0} W^\mu_{\mathbb{F}}(i)$ by Theorem 5.32. Therefore, $W^\nu_{\mathbb{F}}$ and $W^\mu_{\mathbb{F}}$ are in the same block by Lemma 5.36 (and Corollary 2.22). By induction on dominance, $W^\nu_{\mathbb{F}}$ and $W^\gamma_{\mathbb{F}}$ are in the same block, so $W^\mu_{\mathbb{F}}$ and $W^\gamma_{\mathbb{F}}$ are in the same block as we wanted to show.

Finally, suppose that $\nu_{\mathfrak{p}}([h^\mu_{ab}]_{\hat q}) = \nu_{\mathfrak{p}}([h^\mu_{ac}]_{\hat q})$, whenever $(a,b),(a,c) \in [\mu]$. That is, the $\nu_{\mathfrak{p}}$-valuation of the hook lengths in μ are constant along rows. Since μ is not an e-core it has at least one removable e-hook. Let R be the lowest removable e-hook in μ. That is, no other removable e-hook in μ has nodes in a lower row. Since the $\nu_{\mathfrak{p}}$-valuations of the hook lengths are constant on rows, R is contained in a single column of μ. Let σ be the partition obtained by wrapping an e-hook onto the end of the first row of $\mu \backslash R$ and let ν be the partition obtained by wrapping R onto the bottom of the first column of $\mu \backslash R$. Then $\sigma \rhd \mu \rhd \nu$. Moreover, $W^\mu_{\mathbb{F}}$ and $W^\nu_{\mathbb{F}}$ both appear with non-zero coefficient in $\sum_{i>0} W^\sigma_{\mathbb{F}}(i)$ by Theorem 5.32. Therefore, the three Weyl modules $W^\sigma_{\mathbb{F}}$, $W^\mu_{\mathbb{F}}$ and $W^\nu_{\mathbb{F}}$ all belong to the same block by Lemma 5.36. By induction $W^\nu_{\mathbb{F}}$ and $W^\gamma_{\mathbb{F}}$ are in the same block since $\mu \rhd \nu$. Hence, $W^\mu_{\mathbb{F}}$ and $W^\gamma_{\mathbb{F}}$ are in the same block, completing the proof.

Chapter 6

Page 98, Proposition 6.1: It is not immediate that the map defined in the proof of the proposition is an $\mathscr{H}_{n-1}$-module homomorphism. The following expanded argument establishes this.

By Proposition 3.22, $\{\, m_{\mathfrak{t}} \mid \mathfrak{t} \in \operatorname{Std}(\lambda) \,\}$ is a basis of S^λ. Furthermore, by Corollary 3.4 and Corollary 3.21, if $\mathfrak{t} \in \operatorname{Std}(\lambda)$ and $h \in \mathscr{H}_{n-1}$ then $m_{\mathfrak{t}} h$ is a linear combination of terms $m_{\mathfrak{v}}$ where $\mathfrak{v}$ is a standard λ–tableau and n appears in either the same row or a later row of $\mathfrak{v}$ than it does in $\mathfrak{t}$; that is, $\operatorname{Shape}(\mathfrak{v}{\downarrow}(n-1)) \trianglerighteq \operatorname{Shape}(\mathfrak{t}{\downarrow}(n-1))$. Furthermore, by definition, $\operatorname{Shape}(\mathfrak{t}{\downarrow}(n-1)) \longrightarrow \lambda$.

For $i = 1, 2, \dots, z$ let $\operatorname{Std}_i(\lambda) = \{\, \mathfrak{t} \in \operatorname{Std}(\lambda) \mid \operatorname{Shape}(\mathfrak{t}{\downarrow}(n-1)) = \nu_i \,\}$. Then these sets partition $\operatorname{Std}(\lambda)$ and the map $\mathfrak{t} \longmapsto \mathfrak{t}{\downarrow}(n-1)$ defines a bijection

$$\operatorname{Std}(\lambda) = \coprod_{i=1}^{z} \operatorname{Std}_i(\lambda) \xrightarrow{\ \simeq\ } \coprod_{i=1}^{z} \operatorname{Std}(\nu_i).$$

For $1 \le i \le z$, define $S^{(i)} = S^{(i)}_R$ to be the R–module of $\operatorname{Res} S^\lambda$ with basis $\{\, m_{\mathfrak{t}} \mid \mathfrak{t} \in \operatorname{Std}_j(\lambda) \text{ for } i \le j \le z \,\}$. By the first paragraph, each $S^{(i)}$ is an $\mathscr{H}_{n-1}$–submodule of S^λ. To complete the proof it is enough to show that $S^{\nu_i} \cong S^{(i)}/S^{(i+1)}$, for $1 \le i \le z$. Define $\theta_i : S^{(i)}/S^{(i+1)} \longrightarrow S^{\nu_i}$ to be the R-linear map determined by

$$\theta_i(m_{\mathfrak{t}} + S^{(i+1)}) = m_{\mathfrak{t}\Downarrow(n-1)}, \qquad \text{for } \mathfrak{t} \in \operatorname{Std}_i(\lambda).$$

By definition, θ_i is an isomorphism of R-modules. We will show that θ_i is an isomorphism of $\mathscr{H}_{n-1}$-modules.

Recall that $\mathcal{Z} = \mathbb{Z}[\hat q, \hat q^{-1}]$ and that $S^\mu_R \cong S^\mu_{\mathcal{Z}} \otimes_{\mathcal{Z}} R$ for any partition μ. By definition, the submodules $S^{(i)}_R$ of S^λ_R are R-free and $S^{(i)}_R \cong S^{(i)}_{\mathcal{Z}} \otimes_{\mathcal{Z}} R$. Therefore, in order to show that θ_i is an $\mathscr{H}_{n-1}$-module homomorphism it is enough to consider the case when $R = \mathcal{Z}$. Let $\mathcal{K} = \mathbb{Q}(\hat q)$ be the field of fractions of $\mathcal{Z}$. Then $S^\lambda_{\mathcal{K}} \cong S^\lambda_{\mathcal{Z}} \otimes_{\mathcal{Z}} \mathcal{K}$, so by considering $S^\lambda_{\mathcal{Z}}$ as a $\mathcal{Z}$-submodule of $S^\lambda_{\mathcal{K}}$ it follows that θ_i is an $\mathscr{H}_{\mathcal{Z},\hat q}(\mathfrak{S}_{n-1})$-module homomorphism if and only if $\theta_i \otimes 1_{\mathcal{K}}$ is a $\mathscr{H}_{\mathcal{K},\hat q}(\mathfrak{S}_{n-1})$-module homomorphism. Hence, we are reduced to the case when $R = \mathcal{K}$.

Let $\{\, f_{\mathfrak{t}} \mid \mathfrak{t} \in \operatorname{Std}(\lambda) \,\}$ be the seminormal basis of $S^\lambda_{\mathcal{K}}$ as defined in Theorem 3.36. By definition, $f_{\mathfrak{t}} = m_{\mathfrak{t}} F_{\mathfrak{t}}$ for $\mathfrak{t} \in \operatorname{Std}(\lambda)$. Similarly, the Specht modules S^ν have seminormal bases whenever $\nu \longrightarrow \lambda$. Define a new homomorphism $\theta : S^\lambda \longrightarrow \bigoplus_{j=1}^z S^{\nu_j}$ by $\theta(f_{\mathfrak{t}}) = f_{\mathfrak{t}\Downarrow(n-1)}$, for $\mathfrak{t} \in \operatorname{Std}(\lambda)$. By definition, θ is a vector space isomorphism and, in view of Theorem 3.36, it is an $\mathscr{H}_{n-1}$-module homomorphism. Now, by Proposition 3.35, $f_{\mathfrak{t}} = m_{\mathfrak{t}} + \sum_{\mathfrak{u} \rhd \mathfrak{t}} a_{\mathfrak{u}} m_{\mathfrak{u}}$, for some $a_{\mathfrak{u}} \in \mathcal{K}$. Therefore, $S^{(i)}_{\mathcal{K}}$ has basis $\{\, f_{\mathfrak{t}} \mid \mathfrak{t} \in \operatorname{Std}_j(\lambda) \text{ for } i \le j \le z \,\}$ and, moreover, $\theta(S^{(i)}_{\mathcal{K}}) = S^{(i)}_{\mathcal{K}}$. Consequently, θ induces a well-defined map from $S^{(i)}_{\mathcal{K}}/S^{(i+1)}_{\mathcal{K}}$ to $S^{\nu_i}_{\mathcal{K}}$, for $1 \le i \le z$. To complete the proof we claim that $\theta_i(a + S^{(i+1)}) = \theta(a)$, for all $a \in S^{(i)}$ and $1 \le i \le z$.

Let $\mathfrak{t}$ be the unique standard λ-tableau such that $\mathfrak{t}\downarrow(n-1) = \mathfrak{t}^{\nu_i}$. Then $\mathfrak{t}$ dominates all of the tableaux in $\operatorname{Std}_i(\lambda)$ so $f_\mathfrak{t} \equiv m_\mathfrak{t} \pmod{S_\mathcal{K}^{(i+1)}}$. Hence,

$$\theta\big(m_\mathfrak{t} + S_\mathcal{K}^{(i+1)}\big) = \theta\big(f_\mathfrak{t} + S_\mathcal{K}^{(i+1)}\big) = f_{\mathfrak{t}^{\nu_i}} = m_{\mathfrak{t}^{\nu_i}} = \theta_i\big(m_\mathfrak{t} + S_\mathcal{K}^{(i+1)}\big).$$

Now let $\mathfrak{s}$ be an arbitrary tableau in $\operatorname{Std}_i(\lambda)$. Set $d = d(\mathfrak{s}\downarrow(n-1)) \in \mathfrak{S}_{n-1}$. Then d is the unique permutation in $\mathfrak{S}_{n-1}$ such that $\mathfrak{s}\downarrow(n-1) = \mathfrak{t}^{\nu_i}d$, with the lengths adding. Moreover, $\mathfrak{s} = \mathfrak{t}d$ and $m_\mathfrak{s} = m_\mathfrak{t}T_d$. Therefore, since θ is an $\mathscr{H}_{n-1}$-module homomorphism,

$$\begin{aligned}\theta\big(m_\mathfrak{s} + S_\mathcal{K}^{(i+1)}\big) &= \theta\big(m_\mathfrak{t}T_d + S_\mathcal{K}^{(i+1)}\big) = \theta\big(m_\mathfrak{t} + S_\mathcal{K}^{(i+1)}\big)T_d \\ &= m_{\mathfrak{t}_i^\nu}T_d = m_{\mathfrak{s}\downarrow(n-1)} = \theta_i\big(m_\mathfrak{s} + S_\mathcal{K}^{(i+1)}\big).\end{aligned}$$

Therefore, $\theta_i\big(m_\mathfrak{s} + S_\mathcal{K}^{(i+1)}\big) = \theta\big(m_\mathfrak{s} + S_\mathcal{K}^{(i+1)}\big)$ for all $\mathfrak{s} \in \operatorname{Std}_i(\lambda)$. So, θ_i is an $\mathscr{H}_{n-1}$-module homomorphism and the proof is complete.

Page 105, line -5: "good" should be "removable".

Page 119, line 20: delete not.

Page 128, paragraph -1: The paper of Martin and Russel contained a serious gap and even missed some cases. Fortunately, Matthew Fayers has now proved that the decomposition numbers of the blocks of weight 3 are at most 1 when $p > 3$; see "Decomposition numbers for weight three blocks of symmetric groups and Iwahori-Hecke algebras", *Trans. Amer. Math. Soc.*, **360** (2008), 1341–1376. In a sequel to this paper, Fayer's proved James' conjecture for blocks of the Iwahori-Hecke algebra of weight 4. See Fayers's paper "James's conjecture holds for weight four blocks of Iwahori-Hecke algebras", *J. Algebra*, **317** (207), 593–633.

Page 135, Note added in proof: Grojnowski and Vazirani [C,E] prove Conjecture 6.54 only up to a permutation of the multipartitions which index the simple modules of the Ariki–Koike algebras. Ariki has now proved that this permutation is trivial, thus establishing Conjecture 6.54. See Ariki's paper "Proof of the modular branching rule for cyclotomic Hecke algebras", *J. Algebra*, **306** (2006), 290–300.

Grojnowksi's [B] proof of the classification of the blocks of the Ariki–Koike algebras (conjecture 6.53) was incomplete. The conjecture has now been proved by Lyle and Mathas; see "Blocks of cyclotomic Hecke algebras", *Adv. Math.*, **216** (2007), 854–878.

Page 135: line 11: A should be B.

Page 135: line -11: E should be D.

APPENDIX A

Page 142, line -11: $\pi^G : M \longrightarrow N$.

Page 142, line 2: The ideals $\operatorname{Rad} P_i \oplus \bigoplus_{j\neq i} P_j$ are only some of the maximal ideals of A. The argument should be replaced with the following.

Write $A = P_1 \oplus \cdots \oplus P_k$ as a direct sum of principal indecomposable modules. Then the modules $\operatorname{Rad} P_i \oplus \bigoplus_{j\neq i} P_j$, for $i = 1, 2, \dots, k$, are maximal ideals of A. The intersection of these ideals is $\operatorname{Rad} P_1 \oplus \cdots \oplus \operatorname{Rad} P_k = \operatorname{Rad} A$; hence, the Jacobson radical of A is contained in $\operatorname{Rad} A$.

Conversely, suppose that M is a maximal ideal of A. Then A/M is simple so $(\operatorname{Rad} A)(A/M)$ is either 0 or A/M. However, $\operatorname{Rad} A$ is nilpotent so $(\operatorname{Rad} A)^n = 0$ for some n; therefore, $(\operatorname{Rad} A)(A/M) = 0$ since otherwise $A/M = 0$. In particular, this shows that $\operatorname{Rad} A \subseteq (\operatorname{Rad} A)A \subseteq M$. Therefore, $\operatorname{Rad} A$ is contained in every maximal ideal of A; consequently, $\operatorname{Rad} A$ is contained in the Jacobson radical of A. This completes the proof.

Please let me know if you find any other problems — Andrew Mathas