Errata for “Iwahori–Hecke algebras and Schur algebras of the symmetric group”

Please let me know if you find any other problems — Andrew Mathas

With thanks to: Meinolf Geck, Darij Grinberg, Sinéad Lyle, Eric Marberg, Liron Speyer, and many others.

CHAPTER 1

Page 3, line 2: To prove that $t$ has a reduced expression of the form $t = s_i \ldots s_{i-1} s_i s_{i-1} \ldots s_i$, take sine work. On the other hand, we need below is that $t \in N(t)$, which is immediate from the definition of $N(t)$ since $t^2 = 1$.

Page 6, proof of Theorem 1.13: The first displayed equation in the proof should read:

\[
\theta_i(e_w) = \begin{cases} 
   e_{s_i w}, & \text{if } \ell(s_i w) > \ell(w), \\
   qe_{s_i w} + (q-1)e_w, & \text{if } \ell(s_i w) < \ell(w).
\end{cases}
\]

Page 8, proof of Proposition 1.16: In the first displayed equation, the exponent of $q$ is missing a bracket and should read $q^{\frac{1}{2}(\ell(x)+\ell(y)-\ell(xy))}$.

Page 9, Corollary 1.17: $\mathcal{K}$ should be the algebraic closure of $\mathbb{C}(q)$

Page 10, proof of Theorem 1.18: $\mathcal{H}_a$ should be defined as the $\mathbb{R}$-submodule with basis $\{ T_w | 1 \leq b \leq a \}$.

Page 11, Exercise 3: Part (i) requires that $n$ is invertible in $\mathbb{R}$. In part (ii) $\mathbb{R}$ should be $\mathbb{R}$.

Page 11, Exercise 4: Add: where $\mathcal{K} \equiv \mathbb{Q}(q)$. In part (ii), $V''$ is a subspace of $V_q$.

CHAPTER 2

Page 16, Example 2.2 (i): $\hat{A}^n = x^{n+1} R[x]$ is the set of polynomials of minimal degree greater than $n$.

Page 17, line -2: Replace “...and one can check that $C^+ \cong \text{Hom}_R(C^\lambda, R)$” with “and $\text{Hom}_R(C^+\lambda, R)$ is a right $A$-module with $A$-action given by $(f \cdot a)(x) = f(ax)$, for $f \in \text{Hom}_R(C^\lambda, R), a \in A$ and $x \in C^+\lambda$.”

Page 18, §2.10: “maximal ideals” should be “maximal submodules”.

Page 21, proof of 2.18: Delete “that $C^v \cong \text{Hom}_R(C^{+v}, R)$ as right $A$-modules. Therefore” and after the displayed equation add “Hence, dim $P^\lambda \otimes_A C^{v \lambda} = [\text{Hom}_R(C^{+v}, R) : D^\lambda] = d_{v, \lambda}$ since $\text{Hom}_R(D^{v \mu}, R) \cong D^v$ whenever $D^v \neq 0$.”

Page 24, exercise 7(i). Delete “In addition, show that $C^\lambda \cong \text{Hom}_R(C^*\lambda, R)$ for all $\lambda \in \Lambda$.

Page 25, line 3: change parenthetical remark to (the four Kazhdan–Lusztig bases $\{C_x, C'_x, D_x\}$ and $\{D'_x\}$ of $\mathcal{H}(\mathfrak{S}_n)$ are all cellular; however, this is not true for Hecke algebras of other types).

Page 28, line -12: $\mathcal{H}(\mathfrak{S}_n) \cong \mathcal{H}(\mathfrak{S}_{n_1}) \otimes \cdots \otimes \mathcal{H}(\mathfrak{S}_{n_k})$.

CHAPTER 3

Page 30, line -5: $\ell(w_1 \ldots t_j) \leq \ell(w) - j$, for $j = 1, 2, \ldots, k$.

Page 38, Warning: $S^\lambda$ is the dual of the Dipper–James Specht module indexed by $\lambda$; that is, $S^\lambda \cong \langle S^\lambda_{-1} \rangle^\circ$. One can check that $\langle S^\lambda_{-1} \rangle^\circ \cong S^\lambda_{1}$ so it is necessary to replace $\lambda$ with $\lambda'$ when comparing our results with those of Dipper and James.

Page 41, line -1: The reduction to the case where $k = n$ is a bit of a leap. Here are more details.

Suppose $k < n$ where $k$ is the number in the last row and the first column of $t^\lambda$. As in the second paragraph of the proof, let $\mu = (\lambda_1, \ldots, \lambda_r, 1)$. Then $k = |\mu|$ and $m_\lambda = h_{\mu n} m_\mu$ if we set $h = \sum w T_w$, where the sum is over the elements of $\mathfrak{S} \{ (k, k+1, \ldots, n) \}$. By the argument for the case when $k = n$ it follows that

\[ m_\lambda L_k = h m_\mu L_k = h \left ( \text{res}_{\mu} (k) m_\mu + \sum_{u, v \in \mathfrak{S} \{ (k, k+1, \ldots, n) \}} r_{uv} m_{uv} \right ), \]

for some $r_{uv} \in R$. If $\nu$ is a partition of $k$ let $\hat{\nu}$ be the partition of $n$ obtained by appending $1^{n-k}$ to $\nu$ and if $u \in \mathfrak{S} \{ (k, k+1, \ldots, n) \}$ let $\hat{u}$ be the unique $\hat{\nu}$-tableau such that $u \downarrow k = u$. Under the natural embedding $\mathcal{H}(\mathfrak{S}_k) \hookrightarrow \mathcal{H}(\mathfrak{S}_n)$, it is easy to see that if $\nu \sqsupset \mu$ and $u, v \in \mathfrak{S} \{ (k, k+1, \ldots, n) \}$ then $m_{uv} = m_{\hat{u} \hat{v}}$. Notice that if $\nu \sqsupset \mu$ then $\hat{\nu} > \lambda$. Therefore,
returning to the last displayed equation,
\[ m_\lambda(L_k - \text{res}_U(k)) = h \sum_{\nu \supset \mu \atop u,v \in \text{Std}(\nu)} r_{uv} m_{uv} \in H_\lambda \cap \bigcap_{\nu' > \lambda} H_{\nu'} \subseteq H_\lambda. \]

Therefore, \( m_\lambda L_k \equiv \text{res}_U(k)m_\lambda \pmod{H_\lambda} \) as we needed to show.

Page 44, line -6: missing bracket
Page 46, line 11: Replace \( t^i_\nu \) with \( l_e \) throughout.
Page 46, Lemma 3.40: The proof of this lemma is not particularly clear. Here is a better argument:

Let \( L_0, L_1, \ldots, L_k \) be the ladders which meet \( \lambda \) and let \( r_\alpha \) be the row index of the highest node in \( L_\alpha \cap \lambda \), for \( 0 \leq \alpha \leq k \). By definition, if \( b \geq 1 \) then \( \lambda_b - \lambda_{b+1} < \epsilon \) because \( \lambda \) is \( \epsilon \)-restricted. Therefore, if \( (i,j) \) and \( (i',j') \) are two nodes in the ladder \( L_k \), for \( k = L_\nu(i,j) \), then \( (i',j') \in \lambda \) whenever \( (i,j) \in \lambda \) and \( i \leq i' \). That is, the ladder \( L_0, \ldots, L_k \) are ‘unbroken’ in the sense that all of the nodes which belong to the same ladder as \( (i,j) \) and appear in a later row also belong to \( \lambda \). Hence, we see that \( r_0 \geq r_1 \geq \cdots \geq r_k \).

Now suppose that \( t \) is a standard tableau, not necessarily of shape \( \lambda \), such that \( \text{res}(t) = \text{res}(t^i_\nu) \). To prove the Lemma we have to show that \( t \geq t^i_\nu \). We argue by induction on \( n \).

When \( n = 1 \) then \( t = t^i_\nu = t^i_\nu \) and there is nothing to prove, so suppose that \( n > 1 \). Let \( i = \text{res}_U(n) \) and let \( m \) be maximal such that \( \text{res}_U(m) \neq i \). Thus, recalling that \( L_0, L_1, \ldots, L_k \) are the ladders which meet \( \lambda \), \( n \) appears at the top of the ladder \( L_k \) in \( t^i_\nu \) and \( m \) appears at the top of \( L_{k-1} \). Let \( \mu = \text{Shape}(t^i_\nu \downarrow m) \).

As noted above, the ladders \( L_0, \ldots, L_k \) are unbroken. Therefore, \( \mu \) is an \( \epsilon \)-restricted partition and \( t^i_\nu = t^i_\nu \downarrow m \).

Moreover, \( t \downarrow m \) is a standard tableau with residue sequence \( \text{res}(t^i_\nu) \). Therefore, by induction on \( n \),

\[
(1) \quad \text{Shape}(t \downarrow r) \supset \text{Shape}(t^i_\nu \downarrow r) = \text{Shape}(t^i_\nu \downarrow r),
\]

for \( r = 1, 2, \ldots, m \), with equality throughout if and only if \( t \downarrow m = t^i_\nu \). Consequently, if \( t \not\geq t^i_\nu \) then there must exist an integer \( p \) such that \( \text{Shape}(t \downarrow p) \not\supset \text{Shape}(t^i_\nu \downarrow p) \) and \( m < p \leq n \). Let \( p \) be minimal with this property and set \( \nu = \text{Shape}(t \downarrow p) \).

By construction, \( p \) appears in row \( r_k + n - p \) of \( t^i_\nu \). Therefore, by the minimality of \( p \),

\[
\nu_1 + \cdots + \nu_{k+n-p} < \lambda_1 + \cdots + \lambda_{k+n-p} = \mu_1 + \cdots + \mu_{k+n-p} + 1.
\]

Since \( t \downarrow m \geq t^i_\nu \) this forces \( \nu_1 + \cdots + \nu_{k+n-p} = \mu_1 + \cdots + \mu_{k+n-p} \) and hence that \( t \downarrow m = t^i_\nu \).

As \( m + 1, \ldots, n \) occupy the lowest addable \( i \)-nodes in \( t^i_\nu \) this easily implies that \( t \geq t^i_\nu \), giving a contradiction and completing the proof.

Page 47, line 4: \( \lambda \) is a composition of \( \lambda \).
Page 47, line 5: \( g_l \) is the greatest common divisor of \( \{ [\gamma]_l^q \in Z \mid \gamma \supset \lambda \} \). That is, we must define \( g_l \) to be an element of the ring \( Z = Z[q, q^{-1}] \).
Page 47, Lemma 3.42: Suppose that \( R = Z \), that \( \lambda = (\lambda_1, \ldots, \lambda_k) \) is a partition of \( n \),
Page 47, line -7: On the other hand, if \( \lambda \geq \nu \) then \( (\lambda_1, \lambda_2, \ldots, \lambda_k) \geq (\lambda_1, \lambda_1 - \mu_1, \ldots, \lambda_k - \mu_k) \); ...
Page 49, Exercise 2: Let \( \lambda \) and \( \mu \) be partitions of \( n \).
Page 50, exercise 18(ii): As pointed out to me by Eric Marberg, it is not true that \( G_\mu \) is a subgroup of the column stabiliser of \( \psi \cdot d \); for example, take \( \lambda = \mu = (2, 2, 1) \) and \( d = (2, 3, 4, 5) \). Instead one should prove that \( i \) and \( j \) are in different rows of \( \psi \cdot d \) whenever they are in the same row of \( \psi \cdot d \). This implies that \( \mu' \geq \lambda \).
Page 53, line -7: \( h_i = h_i^\lambda \).

**CHAPTER 4**

Page 58, line 4: last(T) = \[ \begin{array}{cccc}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
9 & 10 & 11 & 12 \\
13 & 14 & 15 & 16 \\
\end{array} \]

**CHAPTER 5**

Page 70, line 5: if and only if \( d(\text{first}(S)) \geq d(\text{first}(T)) \).
Page 72, line 2: then \( a < c \)
Page 98, Proposition 6.1: It is not immediate that the map defined in the proof of the proposition is an

Page 94, Conjecture 4.7(iii): This should read: \( \nu \)

Page 81, line 19: smallest should be largest

Page 78, line -2: \( x \)

Page 73, line 18: Note that if \( \lambda \) appears with non-zero multiplicity in the row standard but not standard. As in the last paragraph, \( m_1m_\mu = \sum_\sigma c_\sigma m_\sigma \) and the sum is over the row standard \( \lambda \)-tableau \( \mathbf{v} \) such that \( \mu(\mathbf{v}) = T \). Now if \( \mathbf{v} \) is row standard and \( \mu(\mathbf{v}) = T \) then \( \mathbf{v} \triangleright \text{last}(T) \), so if

\[
m_1m_\mu = \sum_{\sigma \in \text{Std}(\lambda)} c_\sigma m_\sigma, \quad \text{for } c_\sigma \in \mathbb{R},
\]

then \( c_\sigma \neq 0 \) only if \( \mathbf{v} \triangleright \text{last}(T) \) by Lemma 3.15. Note that if \( c_\sigma \neq 0 \) then \( \sigma \) strictly dominates \( \text{last}(T) \) because \( \text{last}(T) \) is not standard. On the other hand, by Proposition 4.14, \( m_1m_\mu \) belongs to the span of \( \{ m_\sigma \mid \sigma \in \mathcal{T}_0(\lambda, \mu) \} \), so \( c_\sigma \neq 0 \) only if \( \mu(\sigma) = \sigma \in \mathcal{T}_0(\lambda, \mu) \). It follows that \( m_1m_\mu = \sum_\sigma c_\sigma m_\sigma \), where \( c_\sigma \neq 0 \) only if \( \sigma \triangleright \text{first}(T) \). Hence, (ii) follows.

Page 75, line 1: Suppose that \( t = \text{first}(T) \). Then \( \pi^\mu_1 \varphi_T = \varphi_1 \varphi_{T^\mu} \ldots \)

Page 78, line -2: \( B_3 \) should be \( G_4 \).

Page 81, line 19: smallest should be largest

Page 81, line -15: \( M_\varphi(k) = M(k) \otimes_R F \) should be \( M_\varphi(k) = (M(k) + pM(k))/pM(k) \).

Page 83, line 8: \( W^\lambda_\varphi \varphi_{T=\varphi^T} \) should be \( W^\lambda_\varphi \varphi_{T=\varphi^T}(i) \)

Page 85, line 10: \( \lambda \) is well-defined

Page 86, Proof of Theorem 5.37: The last paragraph of the proof contains a gap. The proof breaks down in the case where \( W^\lambda \) is irreducible. The following argument corrects this.

To prove the theorem it is enough to show that \( W^\mu_\varphi \) and \( W^\nu_\varphi \) are in the same block. If \( \nu_\varphi([h^\mu_\varphi]_q) \neq \nu_\varphi([h^\nu_\varphi]_q) \), for some nodes \( (a, b), (a, c) \in [\mu] \), then we may assume that \( b = 1 \) and let \( \nu \) be the partition obtained by removing the \( (a, c) \)-hook from \( \mu \) and wrapping it back on the bottom of the first column of \( \mu \setminus R_{ac}^\mu \). Then \( \mu \triangleright \nu \) and \( W^\nu_\varphi \) appears with non-zero multiplicity in \( \sum_{i > 0} W^\nu_\varphi(i) \) by Theorem 5.32. Therefore, \( W^\mu_\varphi \) and \( W^\nu_\varphi \) are in the same block by Lemma 5.36 (and Corollary 2.22). By induction on dominance, \( W^\nu_\varphi \) and \( W^\nu_\varphi \) are in the same block, so \( W^\mu_\varphi \) and \( W^\nu_\varphi \) are in the same block as we wanted to show.

Finally, suppose that \( \nu_\varphi([h^\mu_\varphi]_q) = \nu_\varphi([h^\nu_\varphi]_q) \), whenever \( (a, b), (a, c) \in [\mu] \). That is, the \( \nu_\varphi \)-valuation of the hook lengths in \( \mu \) is constant along rows. Since \( \mu \) is not an \( e \)-core it has at least one removable \( e \)-hook. Let \( R \) be the lowest removable \( e \)-hook in \( \mu \). That is, no other removable \( e \)-hook in \( \mu \) has nodes in a lower row. Since the \( \nu_\varphi \)-valuations of the hook lengths are constant on rows, \( R \) is contained in a single column of \( \mu \). Let \( \sigma \) be the partition obtained by wrapping an \( e \)-hook onto the end of the first row of \( \mu \setminus R \) and let \( \nu \) be the partition obtained by wrapping \( R \) onto the bottom of the first column of \( \mu \setminus R \). Then \( \sigma \triangleright \mu \triangleright \nu \). Moreover, \( W^\mu_\varphi \) and \( W^\nu_\varphi \) both appear with non-zero coefficient in \( \sum_{i > 0} W^\nu_\varphi(i) \) by Theorem 5.32. Therefore, the three Weyl modules \( W^\mu_\varphi \), \( W^\mu_\varphi \) and \( W^\nu_\varphi \) all belong to the same block by Lemma 5.36. By induction \( W^\mu_\varphi \) and \( W^\nu_\varphi \) are in the same block since \( \mu \triangleright \nu \). Hence, \( W^\mu_\varphi \) and \( W^\nu_\varphi \) are in the same block, completing the proof.

Page 94, Conjecture 4.7(iii): This should read: \( \nu_{e, \varphi}(h_{\lambda_0}^\lambda) = -1 \) for \( 1 \leq a \leq k \) and \( 1 \leq b \leq l \).

CHAPTER 6

Page 98, Proposition 6.1: It is not immediate that the map defined in the proof of the proposition is an \( H_{n-1} \)-module homomorphism. The following expanded argument establishes this.

By Proposition 3.22, \( \{ m_\lambda \mid \lambda \in \text{Std}(\lambda) \} \) is a basis of \( S^\lambda \). Furthermore, by Corollary 3.4 and Corollary 3.21, if \( t \in \text{Std}(\lambda) \) and \( h \in H_{n-1} \) then \( m_\lambda h \) is a linear combination of terms \( m_\nu \) where \( \nu \) is a standard \( \lambda \)-tableau and \( n \)
appears in either the same row or a later row of \( n \) than it does in \( t \); that is, \( \text{Shape}(v \downarrow (n - 1)) \supseteq \text{Shape}(t \downarrow (n - 1)) \). Furthermore, by definition, \( \text{Shape}(t \downarrow (n - 1)) \rightarrow \lambda \).

For \( i = 1, 2, \ldots, z \) let \( \text{Std}_i(\lambda) = \{ t \in \text{Std}(\lambda) \mid \text{Shape}(t \downarrow (n - 1)) = \nu_i \} \). Then these sets partition \( \text{Std}(\lambda) \) and the map \( t \mapsto t \downarrow (n - 1) \) defines a bijection

\[
\text{Std}(\lambda) = \bigsqcup_{i=1}^{z} \text{Std}_i(\lambda) \rightarrow \prod_{i=1}^{z} \text{Std}(\nu_i).
\]

For \( 1 \leq i \leq z \), define \( S^{(i)} = S_R^{(i)} \) to be the \( R \)-module of \( \text{Res} S^\lambda \) with basis \( \{ m_\nu | t \in \text{Std}_i(\lambda) \mid \text{Shape}(t \downarrow (n - 1)) = \nu \} \). By the first paragraph, each \( S^{(i)} \) is an \( H_{n-1} \)-submodule of \( S^\lambda \). To complete the proof it is enough to show that \( S^{(i)} \cong S^{(i)}/S^{(i+1)} \), for \( 1 \leq i \leq z \). Define \( \theta_i : S^{(i)}/S^{(i+1)} \longrightarrow S^{(i)} \) to be the \( R \)-linear map determined by

\[
\theta_i(m_\lambda + S^{(i+1)}) = m_{\nu_i(n-1)}, \quad \text{for } t \in \text{Std}_i(\lambda).
\]

By definition, \( \theta_i \) is an isomorphism of \( R \)-modules. We will show that \( \theta_i \) is an isomorphism of \( H_{n-1} \)-modules.

Recall that \( Z = \mathbb{Z}[q, q^{-1}] \) and that \( S_R^\mu \cong S_Z^\mu \otimes_Z R \) for any partition \( \mu \). By definition, the submodules \( S_R^{(i)} \) of \( S_R^\lambda \) are \( R \)-free and \( S_R^{(i)} \cong S_Z^{(i)} \otimes_Z R \). Therefore, in order to show that \( \theta_i \) is an \( H_{n-1} \)-module homomorphism it is enough to consider the case when \( R = Z \). Let \( K = \mathbb{Q}(q) \) be the field of fractions of \( Z \). Then \( S_K^\lambda \cong S_Z^\lambda \otimes_Z K \), so by considering \( S_Z^\lambda \) as a \( Z \)-submodule of \( S_K^\lambda \) it follows that \( \theta_i \) is an \( H_{n-1} \)-module homomorphism if and only if \( \theta_i \otimes 1_K \) is an \( H_{n-1} \)-module homomorphism. Hence, we are reduced to the case when \( R = Z \).

Let \( \{ f_t | t \in \text{Std}(\lambda) \} \) be the seminormal basis of \( S_Z^\lambda \) as defined in Theorem 3.36. By definition, \( f_t = m_t F_t \) for \( t \in \text{Std}(\lambda) \). Similarly, the Specht modules \( S^\nu \) have seminormal bases whenever \( \nu \rightarrow \lambda \). Define a new homomorphism \( \theta : S^\lambda \rightarrow \bigoplus_{i=1}^z S^{(i)} \) by \( \theta(f_t) = f_{\theta_i(t)} \), for \( t \in \text{Std}(\lambda) \). By definition, \( \theta \) is a vector space isomorphism and, in view of Theorem 3.36, it is an \( H_{n-1} \)-module homomorphism. Now, by Proposition 3.35, \( f_t = m_t + \sum_{a \in K} u_a m_a \), for some \( u_\lambda \in K \). Therefore, \( S_K^\lambda \) has basis \( \{ f_t | t \in \text{Std}(\lambda) \} \) for \( i \leq j \leq z \) and, moreover, \( \theta(S_K^{(i)}) = S_K^{(i)} \), \( \forall i \leq j \leq z \). Consequently, \( \theta \) induces a well-defined map from \( S_K^{(i)}/S_K^{(i+1)} \) to \( S_K^\lambda \), for \( 1 \leq i \leq z \). To complete the proof we claim that \( \theta_i(a + S^{(i+1)}) = \theta(a) \), for all \( a \in S^{(i)} \) and \( 1 \leq i \leq z \).

Let \( t \) be the unique standard \( \lambda \)-tableau such that \( (n - 1) \downarrow (n - 1) = t^{(i)} \). Then \( t \) dominates all of the tableaux in \( \text{Std}_i(\lambda) \) so \( f_t \equiv f_{\nu_1} \mod S_K^{(i+1)} \). Hence,

\[
\theta(f_t + S_K^{(i+1)}) = \theta f_t = m_{\nu_i(n-1)} = \theta_1(m_t + S_K^{(i+1)}).
\]

Now let \( s \) be an arbitrary tableau in \( \text{Std}_i(\lambda) \). Set \( d = (n-1) \downarrow (n-1) \in \text{Std}_{n-1} \). Then \( d \) is the unique permutation in \( \text{Std}_{n-1} \) such that \( s \downarrow (n-1) = t \downarrow (n-1) \), with the lengths adding. Moreover, \( s = td \) and \( m_s = m_t T_d \). Therefore, since \( \theta \) is an \( H_{n-1} \)-module homomorphism,

\[
\theta(m_s + S_K^{(i+1)}) = \theta(m_t T_d + S_K^{(i+1)}) = \theta(m_t + S_K^{(i+1)})T_d = m_{\nu_i(n-1)} = \theta_1(m_s + S_K^{(i+1)}).
\]

Therefore, \( \theta_i(m_s + S_K^{(i+1)}) = \theta(m_s + S_K^{(i+1)}) \) for all \( s \in \text{Std}_i(\lambda) \). So, \( \theta_i \) is an \( H_{n-1} \)-module homomorphism and the proof is complete.
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Grojnowski’s [B] proof of the classification of the blocks of the Ariki–Koike algebras (conjecture 6.53) was incomplete. The conjecture has now been proved by Lyle and Mathas; see “Blocks of cyclotomic Hecke algebras”, Adv. Math., 216 (2007), 854–878.

Page 135: line 11: A should be B.
Page 135: line -11: E should be D.

APPENDIX A

Page 142, line -11: $\pi^G : M \rightarrow N$.
Page 142, line 2: The ideals $\text{Rad} P_i \oplus \bigoplus_{j \neq i} P_j$ are only some of the maximal ideals of $A$. The argument should be replaced with the following.

Write $A = P_1 \oplus \cdots \oplus P_k$ as a direct sum of principal indecomposable modules. Then the modules $\text{Rad} P_i \oplus \bigoplus_{j \neq i} P_j$, for $i = 1, 2, \ldots, k$, are maximal ideals of $A$. The intersection of these ideals is $\text{Rad} P_1 \oplus \cdots \oplus \text{Rad} P_k = \text{Rad} A$; hence, the Jacobson radical of $A$ is contained in $\text{Rad} A$.

Conversely, suppose that $M$ is a maximal ideal of $A$. Then $A/M$ is simple so $(\text{Rad} A)(A/M)$ is either 0 or $A/M$. However, $\text{Rad} A$ is nilpotent so $(\text{Rad} A)^n = 0$ for some $n$; therefore, $(\text{Rad} A)(A/M) = 0$ since otherwise $A/M = 0$. In particular, this shows that $\text{Rad} A \subseteq (\text{Rad} A)A \subseteq M$. Therefore, $\text{Rad} A$ is contained in every maximal ideal of $A$; consequently, $\text{Rad} A$ is contained in the Jacobson radical of $A$. This completes the proof.

Page 151, line -1:

$$e_1 = \frac{\mu_i(1)}{|G|} \sum_{g \in G} \mu_i(g)g^{-1}.$$  

APPENDIX B

Page 156: The adjustment matrix entry when $n = 10$ and $e = p = 2$ in the $5 \times 5$ matrix in row $(6, 2, 1^2)$ and column $(4^2, 1^2)$ should be 1. In fact, this entry can be omitted because it is contained in the following $16 \times 16$ matrix.