# Project 1

Title: Inequalities in Geometry and Analysis

Supervisor: Jiakun Liu

**Description:** This project surveys fundamental inequalities in geometry and analysis, including the classical isoperimetric inequality, the Brunn-Minkowski inequality, the Sobolev inequality, the Brascamp-Lieb inequality, the Prékopa-Leindler inequality, and many others. The aim is to explore the connections among these inequalities, study their applications, and investigate new proofs.

Prerequisites: Multi-variable calculus, real analysis, and differential geometry.

#### Project 2

**Title:** Modelling liver disease and risk factors

**Supervisor:** Peter Kim and Joachim Worthington (Daffodil Centre)

**Description:** For people with fatty liver disease, routine monitoring to detect development of late-stage liver disease or cancer can lead to better health outcomes; however, for many this is not necessary as their risk is relatively low. For example, alcohol cessation and weight loss can quickly reduce risk of liver disease and cancer, meaning ongoing monitoring has limited benefits.

The goal of this project will be to analyse existing models of liver disease and adapt them to capture the impact of risk factor changes over time, such as weight gain/loss and alcohol use/cessation. This project would suit someone with experience in differential equations and coding/numerical analysis and an interest in epidemiology/public health.

**Prerequisites:** MATH3X63 Nonlinear ODEs with Applications would help a lot.

#### **Project 3**

Title: Neural Posterior Learning for Bayesian Model Choice

Supervisor: Clara Grazian

**Description:** Bayesian model choice provides a principled framework for comparing competing hypotheses, but its implementation remains challenging when likelihoods are intractable or when the model space is high-dimensional. This project proposes to harness neural networks as flexible, data-driven tools for Bayesian model choice. We will investigate neural architectures that learn low-dimensional, sufficient-like representations of the data for distinguishing between candidate models and estimating posterior model probabilities. The outcomes will establish neural posterior learning as a scalable and robust strategy for Bayesian model choice, bridging statistical rigor with modern machine learning advances.

**Prerequisites:** STAT2011/STAT2911 or any other introduction to statistics and probability.

# **Project 4**

Title: Post-quantum cryptography

Supervisor: Nalini Joshi

**Description:** Mathematical ideas underlie public-key cryptography: a method of conveying protected information on open channels. The most widely used ideas are the Rivest-Shamir-Adleman (RSA) public-key algorithm and elliptic curve cryptography, and they rely on the difficulty of factorising large numbers. The trouble is that when sufficiently powerful quantum computers are available, large integers will be factorised quickly, and so we need new mathematical ideas that will be difficult to decode, even for quantum computers. This project explores two mathematical directions: braid groups and modular lattices.

Prerequisites: Second year mathematical subjects.

# **Project 5**

Title: Dynamical systems in finite fields

Supervisor: Nalini Joshi

**Description:** Mathematical models are usually studied in real or complex number fields. But, on computers, the solutions lie in a finite field. This project considers the properties of important maps in finite fields, such as bounds on the number of iterates,

whether they lie on curves, and related properties. **Prerequisites:** Second year mathematical subjects.

#### **Project 6**

Title: Tubes – their geometry and applications

Supervisor: Haotian Wu

**Description:** The volume of a ball around a point in the three-dimensional space is well-known. What is the volume of a tube around a curve in the three-dimensional space? More generally, what about the volume of a tube around a submanifold of a Riemannian manifold? This project studies the geometry, especially the volume, of tubes and explores the applications of tubes in differential geometry.

**Prerequisites:** Multivariable calculus and linear algebra. A background in differential geometry of curves and surfaces is helpful but not necessary.

# **Project 7**

Title: Data science approaches to biomarker translation in precision medicine

Supervisor: Jean Yang

**Description:** In the age of precision medicine, molecular markers discovered through advanced multi-omics techniques hold great potential for improving clinical decision-making. However, their wider applicability is often limited is often restricted because their substantial drop in performance when applied beyond the original discovery datasets. Such differences may arise from differences in cohort composition or different diagnostic platforms. This project investigates machine learning and AI methods that ensure both the markers (features) and their associated models remain transferable incorporating translational constraints in health care.

**Prerequisites:** DATA2X02.

# **Project 8**

Title: Quantum automorphisms of trees and their representations

Supervisor: Nathan Brownlowe

**Description:** Compact quantum groups are noncommutative analogues of compact groups. More formally, a compact quantum group is a unital C\*-algebra and a coassociative comultiplication satisfying conditions that make commutative examples just function algebras on compact groups. An important class of examples are known as easy quantum groups, and their distinguishing feature is that their representation category is governed by combinatorial structures known as partition diagrams. In this project we will look at examples of quantum groups coming from quantum automorphisms of trees, and we will investigate whether their representations can be described using combinatorial information, as is the case for easy quantum groups.

**Prerequisites:** MATH2X22 and MATH2X23.