INTERNATIONAL CONFERENCE ON NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

10–14 JULY 2023, SYDNEY, AUSTRALIA

About the conference

This conference is being held in honour of Professor Neil Trudinger's 80th birthday to celebrate his important contributions in nonlinear partial differential equations and geometric analysis. The event aims to host several leading mathematicians as well as to provide an opportunity for young researchers and PhD students to get together to present their new works and discuss recent advances in the field, share ideas and generate/strengthen collaborations. The conference will showcase the latest developments on several themes in partial differential equations and their applications, especially those to which Professor Trudinger has made lasting contributions. These include nonlinear elliptic and parabolic equations, calculus of variations, optimal transportation and geometric analysis.

Sponsored by: *Australian Mathematical Sciences Institute (AMSI), Australian Mathematical Society, The Australian National University, The University of New England, The University of Wollongong and The University of Sydney*

ABSTRACTS OF INVITED TALKS

Image comparison and scaling via nonlinear elasticity

John Ball (Heriot-Watt University and Maxwell Institute for Mathematical Sciences, Edinburgh)

Abstract: A nonlinear elasticity model for comparing images is formulated and analyzed, in which optimal transformations between images are sought as minimizers of an integral functional. The existence of minimizers in a suitable class of homeomorphisms between image domains is established under natural hypotheses, and the question of whether for linearly related images the minimization algorithm delivers the linear transformation as the unique minimizer is discussed. This is joint work with Chris Horner.

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Boundary continuity of nonlocal minimal surfaces in domains with singularities

Serena Dipierro (The University of Western Australia)

Abstract: The stickiness phenomenon is one of the most characteristic features of nonlocal minimal surfaces. In this talk, we will present how this property is influenced by corners and singularities of the domain under consideration.

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Fine properties of monotone mappings arising in optimal transport problems for non quadratic costs

Cristian E. Gutiérrez (Temple University)

Abstract: This talk focuses on recent results concerning various properties of mappings arising in optimal transport problems for non quadratic costs. The cost functions considered have the form $c(x, y) = h(x - y)$, where $h \in C^2(\boldsymbol{R}^n)$ is convex, positively homogeneous of degree $p \geq 2$, and $D^2h(x)$ has eigenvalues bounded away from zero and infinity for all $x \in S^{n-1}$. A multivalued mapping $T : \boldsymbol{R}^n \to \mathcal{P}(\boldsymbol{R}^n)$ is c-monotone if $c(\xi, x) + c(\zeta, y) \leq c(\xi, y) + c(\zeta, x)$ for all $\xi \in Tx, \zeta \in Ty$ for all $x, y \in \boldsymbol{R}^n$. Optimal maps with respect to the cost c are c-monotone. If $h(x) = |x|^2$, then c-monotonicity is the standard monotonicity $(\xi - \zeta) \cdot (x - y) \geq 0$ having a large number of applications to optimization and nonlinear evolution pdes.

We prove that c-monotone mappings T are single valued a.e. and establish local L^∞-estimates on balls for $u(x) = Tx - Ax - b$ for each matrix A and each vector b in terms of averages of u on a slightly larger ball. As a consequence, we deduce that T is first order differentiable almost everywhere.

This research is a continuation of recent work in collaboration with Annamaria Montanari (U. of Bologna), originated from recent results by M. Goldman and F. Otto concerning partial regularity of optimal transport maps for the quadratic cost.

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Existence of a self-similar solution to a nonlocal doubly nonlinear parabolic equation on the whole Euclidean space

Daniel Hauer (The University of Sydney)

Abstract: In this talk, I outline the existence and uniqueness of a self-similar solution to a parabolic equation on the whole Euclidean space where the principal part is the nonlocal fractional p-Laplacian composed with a power function. The proof generalizes methods developed by J. L. Vazquez [Nonlinear Anal. (2022) and Calc. Var. Partial Differential Equations (2021)] for the same evolution problem without perturbation. In particular, I present an Aleksandrov symmetry principle, which can be applied to the mild solutions of the evolution equation in L^1 governed by the doubly nonlinear nonlocal operator, and might be of independent interest.

These research results are part of my student's PhD thesis - Timothy Collier.

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Nonlinear PDE and a Stochastic Analysis Approach to in the Parisi Formula in Spin Glass Theory

Elton P. Hsu (Northwestern University)

Abstract: The Parisi variational formula is a fundamental result in spin glass theory. The attendant Parisi functional involves a nonlinear Hamilton-Jacobi type PDE. The upper bound in the Parisi formula is given by Guerra's interpolation method reminiscent of large deviation theory in probability theory. In this talk, I will present a new approach to the upper bound using stochastic analysis applied to the nonlinear PDE. Among the techniques from stochastic analysis we will use include path space integration parts for the Wiener measure (Brownian motion), the Girsanov transform (exponential martingales), and the Feynman-Kac formula. The key observation is that the nonlinear partial differentiation equation figuring in Parisi's variation formula becomes formally linear after differentiating with respect to the interpolation parameter, thus allowing the full strength of stochastic analysis based on Ito's calculus into play. We hope that this approach will shed some lights on the much more difficult lower bound in the Parisi formula, which was first proved by M. Talagrand and subsequently by D. Panchenko using a different approach.

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Monodromy and asymptotics

Nalini Joshi (The University of Sydney)

Abstract: The Fricke family of cubic surfaces $xyz + x^2 + y^2 + z^2 + a_1x + a_2y + a_3z + b = 0$ is famous in algebraic geometry. In this talk, we give an overview of the connection between such surfaces and solutions of the classical Painlevé equations.

Our recent results show that such surfaces are also associated with discrete Painlevé equations, with some surprising twists. In particular, we found Segre surfaces, which are defined by quadric polynomials in $\mathbb{P}^4$. Their continuum limits give Segre surfaces associated with the Painlevé equations.

The results are based on collaborations with Pieter Roffelsen and Marta Mazzocco.

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Dissipative lattice dynamical systems

Peter Kloeden (Eberhard Karls Universität Tübingen)

Abstract: Lattice dynamical systems (LDS) are essentially infinite dimensional systems of ordinary differential equations and are formulated as ordinary differential equations on Banach spaces of bi-infinite sequences. There have been many generalisations to include delayed, random and stochastic terms as well as multi-valued terms.

LDS arise in a wide range of applications with intrinsic discrete structures such as chemical reaction, pattern recognition, image processing, living cell systems, cellular neural networks, etc. Sometimes they are derived as spatial discretisations of models based on partial differential equations, but they need not arise in this way.

There is an extensive list of papers on lattice dynamical systems. During the 1990s there was a strong emphasis on patterns and travelling waves in such systems. In recent decades attention focused on attractors with results summarised in the monograph

Peter Kloeden and Xiaoying Han, *Dissipative Lattice Dynamical Systems*. World Scientific Publishing Co. Inc, Singapore, 2023.

This talk focuses on dissipative lattice dynamical systems and their attractors of various forms such as autonomous, nonautonomous and random. The existence of such attractors is established by showing that the corresponding dynamical system has an appropriate kind of absorbing set and is asymptotically compact in some way.

The main ideas and techniques are discussed here and representative examples are presented including the approximation of Heaviside switching functions in LDS by sigmoidal functions.

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Analysis of singularities of area minimizing currents

Brian Krummel (The University of Melbourne)

Abstract: The monumental work of Almgren in the early 1980s showed that the singular set of a locally area minimizing rectifiable current T of dimension n and codimension ≥ 2 has Hausdorff dimension at most $n-2$. In contrast to codimension 1 area minimizers (for which it had been established a decade earlier that the singular set has Hausdorff dimension at most $n-7$), the problem in higher codimension is substantially more complex because of the presence of branch point singularities, i.e. singular points where one tangent cone is a plane of multiplicity 2 or larger. Almgren's lengthy proof (made more accessible and technically streamlined in the much more recent work of De Lellis–Spadaro) showed first that the non-branch-point singularities form a set of Hausdorff dimension at most $n-2$ using an elementary argument based on the tangent cone type at such points, and developed a powerful array of ideas to obtain the same dimension bound for the branch set separately. In this strategy, the exceeding complexity of the argument to handle the branch set stems in large part from the lack of an estimate giving decay of T towards a unique tangent plane at a branch point.

We will discuss a new approach to this problem (joint work with Neshan Wickramasekera). In this approach, the set of singularities (of a fixed integer density q) is decomposed not as branch points and non-branch-points, but as a set $\mathcal{B}$ of branch points where T decays towards a (unique) plane faster than a fixed exponential rate, and the complementary set $\mathcal{S}$. The set $\mathcal{S}$ contains all (density q) non-branch-point singularities, but a priori it could also contain a large set of branch points. To analyse $\mathcal{S}$, the work introduces a new, intrinsic frequency function for T relative to a plane, called the planar frequency function. The planar frequency function satisfies an approximate monotonicity property, and takes correct values (i.e. ≤ 1) whenever T is a cone (for which planar frequency is defined) and the base point is the vertex of the cone. These properties of the planar frequency function together with relatively elementary parts of Almgren's theory (Dirichlet energy minimizing multivalued functions and strong Lipschitz approximation) imply that T satisfies a key approximation property along $\mathcal{S}$: near each point of $\mathcal{S}$ and at each sufficiently small scale, T is significantly closer to some non-planar cone than to any plane. This property together with a new estimate for the distance of T to a union of non-intersecting planes and the blow-up methods of Simon and Wickramasekera imply that T has a unique non-planar tangent cone at $\mathcal{H}^{n-2}$-a.e. point of $\mathcal{S}$ and that $\mathcal{S}$ is $(n-2)$-rectifiable with locally finite measure. Analysis of $\mathcal{B}$ using the planar frequency function and the locally uniform decay estimate along $\mathcal{B}$ recovers Almgren's dimension bound for the singular set of T in a simpler way, and (again via Simon and Wickramasekera blow-up methods) shows that $\mathcal{B}$ (and hence the entire singular set of T) is countably $(n-2)$-rectifiable with a unique, non-zero multi-valued harmonic blow-up at $\mathcal{H}^{n-2}$-a.e. point of $\mathcal{B}$.

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From Teichmüller to Shoen–Yau: Extremal mappings between Riemann surfaces

Gaven Martin (Massey University)

Abstract: There are two now classical descriptions of the moduli space of a Riemann surface via the theory of extremal mappings. The first from Teichmüller in the 1940s (rigorously established by Ahlfors in 1953) and through the existence of extremal quasiconformal mappings. The second is through Schoen–Yau's existence theory for unique harmonic diffeomorphisms in the 1970s, and developed into a theory of moduli by many, including Wolf, Tromba and Wolpert many years later. The important ingredient in both is the existence of a holomorphic quadratic differential, from the Beltrami coefficient of an extremal quasiconformal mapping (Teichmüller) or from the Hopf equation (Harmonic). These quadratic differentials define the cotangent space to the moduli space. Here we show that in fact both of these approaches are manifestations of the same theory (that of existence of diffeomorphic extremal mappings of finite distortion) in limiting regimes. We identify parameterised families of moduli spaces (Beltrami coefficients) interpolating between these two end cases defined by a parametrised family of degenerate elliptic nonlinear PDEs giving holomorphically parameterised homotopy between the extremal quasiconformal mapping [which is not a diffeomorphism] and the harmonic diffeomorphism.

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Locally Lipschitz product selection in the principal-agent problem

Robert McCann (University of Toronto)

Abstract: The principal-agent problem is an important paradigm in economic theory for studying the value of private information; the nonlinear pricing problem faced by a monopolist is one example; others include optimal taxation, and auction design. For multidimensional spaces of consumers (i.e. agents) and products, with consumers having a (quasi)linear sensitivity to prices and payments, Carlier (2001) reformulated this problem as a maximization over the set of b-convex indirect utility functions, $b(x, y)$ representing the value (or benefit) of product y to agent x. Figalli, Kim, and McCann (2011) derived necessary and sufficient conditions for concavity of this maximization problem by exploiting the Ma-Trudinger-Wang tensor associated to b. When concavity is sufficiently uniform, we prove the agent's choice give a locally Lipschitz map from consumers to products purchased. By adapting an earlier, unpublished, argument of Caffarelli and Lions from the bilinear case $b(x, y) =< x, y >$ to more general quasilinear benefit functions, we construct a competitor which allows us to pinch the maximizing b-convex indirect utility (whose gradient determines this choice) between parabolas. This represents joint work with Cale Rankin and Kelvin Shuangjian Zhang.

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A semigroup approach to the Landau-Lifshitz-Gilbert equation

Jonathan Mui (The University of Sydney)

Abstract: The Landau-Lifshitz-Gilbert (LLG) equation is a fundamental model for the time evolution of magnetisation in a ferromagnet under the influence of an "effective field", which is usually given as the variation of an appropriate energy functional. The basic form of the equation leads to the constraint that the magnetisation vector is of unit length (after rescaling) for all times. This pointwise constraint leads to a quasilinear system of PDEs.

There is now an extensive body of literature on the LLG equation in the case where the energy functional is similar to the Dirichlet energy, so the resulting equations are second order. In this talk, I will discuss a more recently introduced fourth-order LLG equation and consider the basic problem of existence of strong solutions via some classical methods in the theory of semigroups and non-autonomous evolution systems.

This project is work in progress with Ben Goldys.

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Growth theorems as substitution for iteration procedures

Mikhail Safonov (University of Minnesota Twin Cities)

Abstract: We consider a few examples, when so-called growth theorems for second order elliptic and parabolic equations allow to bypass iteration procedures, which are typical in the study of boundary behavior of solutions. On some occasions, they produce explicit estimates for solutions in terms of the given constants, rather than "implicit" dependence by contradiction arguments.

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Hessian estimate for the sigma-2 equation in dimension four

Ravi Shankar (Princeton University)

Abstract: We derive Hessian estimates and interior regularity in four dimensions, and partial regularity in higher than four dimensions, for the sigma-2 equation. Our method also provides respectively Hessian estimates for smooth solutions satisfying a dynamic semi-convexity condition in higher than four dimensions, and a new proof for the corresponding three dimensional results by Warren–Yuan in 2007 without minimal surface methods. The two dimensional result for the sigma-2 or the Monge-Ampere equation was achieved by Heinz in the 1950s. Irregular solutions to the sigma-k with $k > 2$ equations, including the Monge-Ampere equation in higher than two dimensions, were first constructed by Pogorelov in the 1970s.

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A Liouville type theorem for stable minimal hypersurfaces

Leon Simon (Stanford University)

Abstract: The classical Liouville theorem says that a function u which is harmonic on all of $\mathbb{R}^n$ and which is bounded below must be a constant. The talk will present an analogue of this for strictly stable embedded minimal hypersurfaces lying on one side of a cylindrical minimal cone. There will also be a discussion of how this theorem can be applied to construct a rich class of stable minimal hypersurfaces with fractional dimensional singular set.

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On an overdetermined problem involving the fractional Laplacian

Jack Thompson (The University of Western Australia)

Abstract: Overdetermined problems are a type of boundary value problem where 'too many' conditions are imposed on the solution. In general, such a problem is ill-posed, so the main objective is to classify sets in which the problem is well-posed. A classical result due to J. Serrin says that a bounded domain in $\mathbb{R}^n$ that admits a function with constant Laplacian, zero Dirichlet data, and constant Neumann data must be a ball. We consider a semi-linear generalisation of Serrin's problem driven by the fractional Laplacian where the value of the solution is prescribed on surface parallel to the boundary. We prove that the existence of a non-negative solution forces the region to be a ball. We also discuss some further related results. This is joint work with S. Dipierro, G. Poggesi, and E. Valdinoci.

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Some recent progress on Ricci flow

Gang Tian (BICMR and Peking University)

Abstract: In this talk, I will discuss a long-standing problem on type II singularity of Ricci flow. I will report on some recent progress and related results.

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A strict maximum principle for nonlocal minimal surfaces

Enrico Valdinoci (The University of Western Australia)

Abstract: Suppose that two nonlocal minimal surfaces are included one into the other and touch at a point. Then, they must coincide. But this is perhaps less obvious than what it seems at first glance.

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Free boundary problems in the Monge-Ampere equation

Xu-Jia Wang (The Australian National University)

Abstract: There are mainly three free boundary problems related to the Monge-Ampere equation. One is the free boundary problem in the Monge-Ampere obstacle problem, another one is the free boundary in partial optimal transportation, and the third one arises in the Gauss curvature flow of convex hypersurface with flat side. These problems have been studied by many authors, but only the $C^{1,\alpha}$ regularity or the $C^{2,\alpha}$ regularity in dimension two have been obtained. With my collaborators, we proved the $C^{2,\alpha}$ and higher regularities for all the three problems.

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Free boundary mean curvature flow: what works and what doesn't

Valentina Wheeler (The University of Wollongong)

Abstract: In this talk we will give a short overview of the problem and some of the results obtained so far in mean curvature flow with free boundary with an emphasis on the behaviour that was not expected at the time when the setting was proposed. These include the appearance of type 2 curvature singularities on the boundary and loss of graphicality over time.

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Independence of singularity type for long time solutions to the Kähler-Ricci Flow

Hosea Wondo (The University of Sydney)

Abstract: The study of long time singular solutions to the Kähler-Ricci flow has generated much interest due to its relevance to the final step of Song and Tian's analytic minimal model program.

We show that the singularity type of such solutions to the Kähler-Ricci flow does not depend on the initial metric. More precisely, if X is a numerically effective manifold admitting a type III solution to the Kähler-Ricci flow, then any other solution starting from a different initial metric will also be Type III. This generalises a previous result by Y. Zhang for the semi-ample case. This talk is based on joint work with Zhou Zhang.

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On some free boundary value problems arising from subsonic-sonic jet flows and rigidity

Zhouping Xin (Chinese University of Hong Kong)

Abstract: In this talk, I will discuss some results on steady compressible potential jet flows from a finite converging nozzle, which are free boundary problems for a nonlinear degenerate elliptic equation. An important feature is that such problems do not have a variational structure. Formulation of the problems and the existence (and non-existence) of solutions will be discussed. Both finite jets and infinite jets can be obtained by a PDE approach and regularity and properties of the solutions. In particular, a general result on the rigidity of the location of sonic degeneracy will be established. This talk is based on joint works with Chunpeng Wang.

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